

# GRTS – Generalized Random Tessellation Stratified Design

**Proof:** If, for some  $n > 0$ ,  $s$  and  $s + d$  are in the same subquadrant  $Q_{jk}^n$ , then  $f(s)$  and  $f(s + d)$  are in the same interval  $J_m^n$ , so that  $|f(s) - f(s + \delta)| \leq \frac{1}{4^n}$ . The probability that  $s$  and  $s + d$  are in the same subquadrant is the same as the probability of the origin and  $d = (d_x, d_y)$  being in the same cell of a randomly located grid with cells congruent to  $Q_{jk}^n$ . For  $\delta_x, \delta_y \leq \frac{1}{2^n}$ , that probability is equal to  $\frac{|Q^n(0) \cap Q^n(\delta)|}{|Q^n(0)|} = 1 - 2^n(\delta_x + \delta_y) + 4^n \delta_x \delta_y$  where  $Q^n(x)$  denotes a polygon congruent to  $Q_{jk}^n$  centered on  $x$ . For  $D(s, \delta) = |f(s) - f(s + \delta)|$ , then, we have that  $P\left(D \leq \frac{1}{4^n}\right) \geq 1 - 2^n(\delta_x + \delta_y) + 4^n \delta_x \delta_y$ . Thus, the distribution function  $F_D$  of  $D$  is bounded below by

$$F_D(u) \geq \begin{cases} 0, & u \leq \frac{1}{4^n} \\ 1 - 2^n(\delta_x + \delta_y) + 4^n \delta_x \delta_y, & u > \frac{1}{4^n} \end{cases}$$

Everyone - if you are like me and couldn't wait to read about GRTS after our call yesterday... I would really enjoy speaking to any of you that find a successful I&M-related application of this technique. Perhaps the key to gaining wide acceptance of a statistical technique is to: 1) have Steve Fancy like it, and 2) name it after a hearty traditional meal... Cheers, Shawn

Because  $D$